CPIA PUBLICATION 222

HYBRID ROCKET
INTERNAL BALLISTICS

JANUARY 1972

CHEMICAL PROPULSION INFORMATION AGENCY

by DAVID W. NETZER

HYBRID ROCKET
INTERNAL BALLISTICS

JANUARY 1972

By

DAVID W. NETZER

Naval Postgraduate School

Prepared by CPIA

CONTENTS

A_p – port area

B – thermochemical mass transfer number

B' – blowing parameter

C^* – characteristic exhaust velocity

C_f – skin friction coefficient in presence of blowing

C_{fo} – skin friction coefficient with no blowing

C_H – Stanton number

C_m – specific heat of nonvaporizing components in solid grain

C_p – specific heat

D – port diameter

F – thrust

G – mass flux of gas per unit area = $\dot{m}_g/A_p$

G_o – mass flux per unit area at $X = 0$

h – enthalpy

h_T – total enthalpy (including chemical enthalpy)

h_v – heat of gasification of solid grain

h_{v_b} – binder enthalpy of vaporization

k – thermal conductivity of gas

K – mass fraction of nonvaporizing components in solid grain

K_{f_s} – fuel mass fraction of surface

K_{ox_e} – oxidizer mass fraction at boundary layer edge

L – length of grain

Le – Lewis number

$\dot{m}_g$ – mass flow rate

$\dot{m}_f$ – mass flux from solid grain

n – number density of particles in gas

O/F – oxidizer-to-fuel ratio, ν

p – pressure

Pr – Prandtl number

$\dot{Q}_c$ – convective heat transfer with no radiation

$\dot{Q}_r$ – radiative heat transfer

$\dot{Q}_w$ – heat flux to solid

$\dot{r}$ – regression rate of solid grain

Re_x – Reynolds number, $\rho_e u_e X/\mu_e = GX/\mu_e$

t – time

T_o – temperature deep in grain

T_r – effective radiation temperature

T_s – surface temperature

u – X component of velocity

v – y component of velocity

X – axial distance from head-end

y – distance normal to grain

z – radiation path length

α – empirical radiation parameter

δ – boundary layer thickness

Δh – sensible enthalpy difference between flame and wall

Δh_T – total enthalpy difference between flame and wall (including chemical enthalpy)

ΔT – $\left(T_s - T_o\right)$

ε – emissivity

η_b – dimensionless flame height y_b/δ

μ – viscosity

ν – oxidizer-to-fuel ratio, O/F

ρ – density

ρ_v – bulk density of vaporizing component

$\left(\dfrac{\bar{\rho}}{\rho_e}\right)^{0.6}$ – variable--property correction

σ – Stefan-Boltzmann constant

γ – shear stress

ϕ – velocity ratio, u/u_e

Ω – stoichiometry parameter, $K_{fs}\nu/K_{ox_e}$

Subscripts

b – conditions at flame

e – conditions at boundary layer edge

f – solid grain

g – gas

r – in the presence of radiation

w – wall (surface)

ABSTRACT

This report presents a summary of hybrid
rocket internal ballistics. Discussions are
primarily concerned with the current theory
that is based on a heat transfer limited model.
Areas of application, major limitations, kinetic
effects, and controversial aspects are briefly
discussed. Areas are identified that require
additional analytical and/or experimental in-
vestigation.

The review was completed in June 1971, and
does not include bibliographic material after
that date.

FORWARD

The intent of this report is: (a) to present
a bibliography of pertinent literature related to
hybrid rocket internal ballistics, (b) to indicate
which references are the most significant with
regard to the various areas of hybrid combustion
(heat transfer limited model, kinetic effects, de-
sign practices, etc.), (c) to present a brief out-
line of the current internal ballistics theory,
(d) to discuss the areas of application, the major
limitations and controversial aspects of the cur-
rent theory, and (e) to indicate those areas which
require additional experimental and/or analytical
investigation. Literature on hybrid rocket internal
ballistics which had appeared through June 1971 has
been included in this report.

This work was sponsored by the Naval Air
Systems Command, assistant commander for Research
and Technology, Propulsion Administrator, AIR-330
under job order number 4505-1519, 045011-01.

Although there are various types of "hybrid rockets" discussed in the literature, the term normally infers a system consisting of a solid fuel and a gaseous and/or liquid oxidizer. So-called reverse hybrids (solid oxidizer, liquid or gaseous fuel), tribrids, etc., have also received significant attention.

Interest in hybrid rocket propulsion has fluctuated considerably since its inception some thirty-five years ago. However, most of the investigations which have contributed significantly to the current understanding of the internal ballistics of hybrid rockets have been conducted within the last ten years. The majority of the contributions to this field have come from France (ONERA) and the United States.

The hybrid rocket offers some significant advantages. Among these advantages are: deliverable specific impulse as high as that for solid or liquid rockets, high density impulse, space storability, ease of thrust modulation and engine restart, normally free of combustion instability, the ability to provide good performance with smokeless exhaust, and inherent safety. It is the latter characteristic, inherent safety, that has recently revived interest in hybrids. In addition, air-augmented hybrids show promise for future tactical applications.

Many aspects of hybrid technology are similar to those of solid and liquid rocket engines: liquid injectors, tankage, valves, positive expulsion devices, solid grain casting methods, prediction of theoretical performance, and etc.

The one major area of difference is that of internal ballistics. Solid Propellant burning rate is normally characterized by an empirical equation of the form $\dot{r} = ap^n$ and is in many cases sensitive only to pressure and initial grain temperature. A complete understanding of the combustion mechanism(s) of solid propellants is lacking and there is still considerable controversy in this regard. In contrast, the regression rate of the solid grain in a hybrid rocket may be a function of the pressure, the temperature, the grain configuration, and the total mass flux of gas in the port per unit

of cross-sectional area. Thus, predicting hybrid internal ballistics is dependent on an understanding of both combustion and flow phenomena.

A bibliography of hybrid literature was published in 1967 (1)*. In addition, reference Cl lists much of the early experimental work (thru 1962). There also are several good publications which have summarized the state of the art in hybrid internal ballistics and in hybrid rockets in general. Within the last three years several additional publications have appeared which have a significant bearing on the current understanding of combustion phenomena. Although hybrid internal ballistics theory has progressed to the stage where it has been used for the successful design (an subsequent use) of several practical hybrid motors, there remain many significant problem areas which require further investigation. As the utilization of hybrid rockets increases, they will inevitably be required to perform in regimes of operation in which the current internal ballistics theory will be of limited utility.

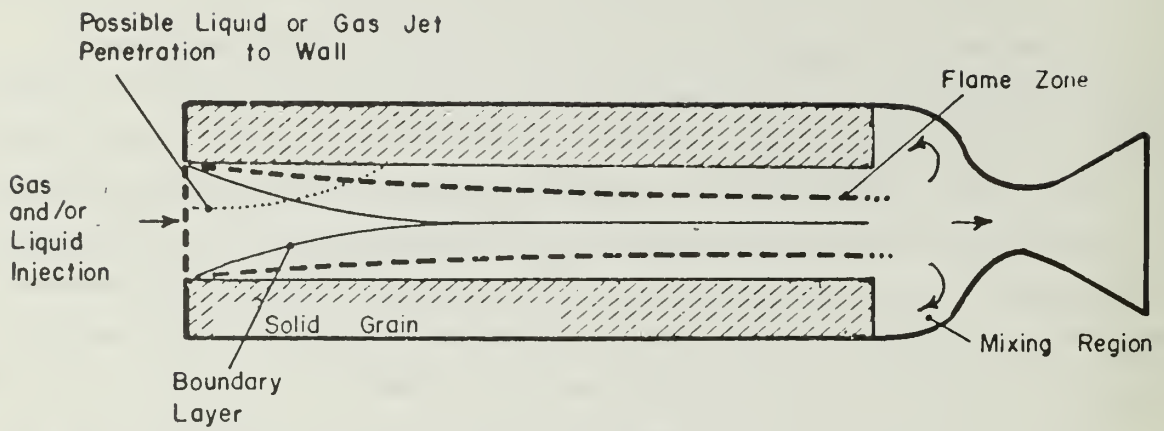*Numbers in parenthesis refer to references.

2.0 GENERAL DISCUSSION

A complete description of the internal ballistics of a combusting system would include: (a) a model for the combustion process; (b) a model for the internal flow (droplet history, boundary layer growth, pressure loss, etc.) which, when coupled with the combustion model, would yield the burning rate (regression rate for solid grains); (c) a description of the combustor (grain geometry, L*, contraction ratio, etc.) and how it would change with time; and (d) a means of predicting the ideal performance. It is the first two aspects that distinguish one system from another.
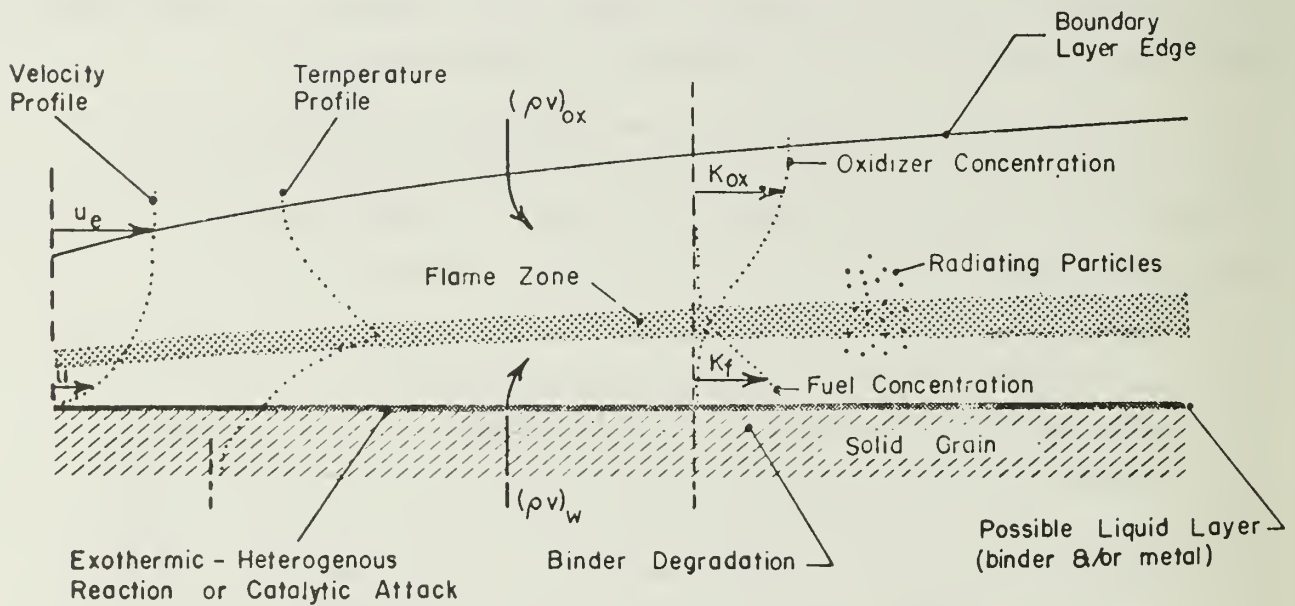
The internal ballistics of hybrid rocket motors are somewhat more complex than those of solid rocket motors. This results not from a lesser understanding of the hybrid system but rather from the fact that the regression rate of the hybrid grain is, in part, dependent upon the total mass flux per unit area passing through the port. This mass flux, in turn, varies with axial position due to the consumption of the solid grain.

An understanding of hybrid internal ballistics is dependent upon an understanding of the interrelationships between the momentum and energy transport and the chemical kinetic mechanisms. A generalized sketch of the processes which may exist in hybrid combustion is shown in Figure 1.

The bulk of hybrid internal ballistics modeling has dealt primarily with the transport of heat by convection and radiation from a diffusion flame zone (located in a turbulent boundary layer) to the solid surface. This transport is complicated by the presence of high rates of wall mass addition. In most of these investigations the wall gasification processes are considered to respond instantaneously to the heat flux and the kinetic processes are neglected. Recent investigations have modified this viewpoint. Other investigators have studied the solid grain regression rate from the alternate approach of the thermal degradation processes of the solid. Neither approach is complete in itself. The former has been more popular because it relates directly to the physical design variables and resulted in a successful engineering model which is applicable to many current hybrid propellant systems. However, future propellants and applications may render the current heat transfer limited model inadequate.

3

Possible Liquid or Gas Jet
Penetration to Wall

Flame Zone

Gas
and/or
Liquid
Injection

Solid Grain

Boundary
Layer

Mixing Region

(a) Overall Schematic

Boundary
Layer Edge

Velocity
Profile

Temperature
Profile

$(\rho v)_{ox}$

u_e

K_{ox}

Oxidizer Concentration

Flame Zone

Radiating Particles

u

K_f

Fuel Concentration

Solid Grain

$(\rho v)_w$

Exothermic – Heterogenous
Reaction or Catalytic Attack

Binder Degradation

Possible Liquid Layer
(binder &/or metal)

(b) Boundary Layer Schematic

Fig. 1   HYBRID ROCKET COMBUSTION PHENOMENA

4

It will be very difficult to handle simultaneous heat and mass transfer together with kinetic mechanisms in a working model for hybrid internal ballistics design. Fortunately, in many current and proposed hybrid systems it appears that the "limiting case" models will be applicable.

Hybrid internal ballistics modeling has yielded good qualitative results and in limiting cases has yielded quite accurate quantitative results. Currently, any quantitative inaccuracies or regimes of operation outside the model assumptions are handled through empirical correction parameters.

Hybrid internal ballistics literature is voluminous and of varied content. Some of it emphasizes the development of analytical models whereas others emphasize the engineering applications. Most of the early (pre 1965) modeling of hybrid combustion has been reviewed by Green (54). Comprehensive development of the current internal ballistics model resulted primarily from investigations at the United Technology Center (UTC). However, other investigators at the Lockheed Propulsion Company (LPC), Rocketdyne, Stanford Research Institute (SRI), and etc. have made pertinent contributions. Some of the more important references are listed here by corporate authors.

LPC: C13 (2 volumes), C14, C15
MIT: 38, 39
ONERA (France): 20, 22, 25, 26
Rocketdyne: 30, 31, 32
SRI: 17
UTC: 3, 4, 5, 7, 8, 10, 12, C4, C5, C6, C10

3.0 BRIEF REVIEW OF HEAT TRANSFER LIMITED MODELS

3.1 Introduction

The material presented in this section is primarily that developed by UTC although reference is also made to other material when it is appreciably different. No attempt has been made to present all of the details and justifications for the model development; these can be found in the cited references. However, some detail is presented to indicate major aspects so that subsequent discussion can be made with regard to the applicability and limitations of the model.

In brief, the model developed in this section has proven to be applicable (although with some reservation) to many of the current hybrid rocket propellant systems and regimes of operation. Thus, the hybrids that can be described reasonably well by the model are those which have fairly simple grain configurations (no sharp corners, no flow mixers within the grain, etc.), which are pure hybrids (little if any solid oxidizer in solid grain), and which operate at relatively high pressures.

The basic model considers the solid phase regression rate to be controlled and limited by convective and radiative heat transfer to the solid grain from a relatively thin diffusion flame in a turbulent boundary layer. The existence of this physical description has been verified by schlieren observations (13) and studies with boundary layer probes (8, 9). All chemical reactions are considered to occur very rapidly in an infinitely thin flame sheet and therefore, all kinetic effects (thermal degradation of binder, heterogenous attack of solid by active oxidizer at the wall, gas phase kinetics, et.) are neglected. The possible existence of a liquid metallic melt on the surface is also neglected in the following development but will be discussed subsequently. The basic model is applicable only to the steady-state and considers the surface to respond to the heat flux instantaneously by decomposing to yield gases (and possibly solids). Thus, in the steady-state

$$\dot{Q}_w = \dot{m}_f h_v = \rho_f \dot{r} h_v = (\rho v)_{wall} h_v \qquad [1]$$

where

h_v = heat of gasification of the solid grain

$\dot{r}$ = regression rate of solid grain

$\dot{Q}_w$ = total heat flux to the surface

ρ_f = density of the solid grain

3.2 Convective Heat Transfer (2, 3, 5, 7, 8, and 12)

The convective heat transfer to the wall can be written

$$\dot{Q}_c = -\left(\frac{k}{\overline{C}_p} \frac{\partial h}{\partial y}\right)_w \qquad [2]$$

where

$\dot{Q}_c$ = convective heat flux to surface

$\overline{C}_p$ = average specific heat of gas

k = thermal conductivity of gas

h = enthalpy

y = distance normal to grain surface

A Stanton number is defined for heat transfer from the flame to the wall in the presence of mass addition such that

$$C_H = \frac{\dot{Q}_c}{\rho_b u_b \Delta h} \qquad [3]$$

where

$$\Delta h = \left(C_p T\right)_b - \left(C_p T\right)_w \qquad [4]$$

Δh = sensible enthalpy difference between the flame and the wall

u_b = velocity at the flame

ρ_b = gas density at the flame

[3] into [1] with $\dot{Q}_c = \dot{Q}_w$ yields

$$\rho_f \dot{r} = \frac{\dot{Q}_c}{h_v} = \frac{\dot{Q}_w}{h_v} = C_H \rho_b u_b \Delta h / h_v \qquad [5]$$

7

An expression is now needed for C_H in the presence of blowing. Empirical expressions for the skin friction coefficient, C_f, in the presence of blowing appear in the literature and it is therefore convenient to determine

$$C_H = f(C_f) \tag{6}$$

Assuming that the Reynolds analogy is applicable between the flame and the wall in the presence of blowing (or Le = 1) yields for constant but nonunity Prandtl number (8, 53)

$$-\frac{\dot{Q}}{\rho \frac{\partial h_T}{\partial y}} = \frac{\tau}{\rho \frac{\partial u}{\partial y}} \, Pr^{-.67} \tag{7}$$

where

h_T = total enthalpy (including chemical enthalpy)

τ = shear stress

Pr = Prandtl number

Le = Lewis number

Near the wall (2, 8)

$$\frac{\dot{Q}}{\dot{Q}_w} = \frac{\tau}{\tau_w} \tag{8}$$

[7] into [8] and integrating between the wall and the flame yields

$$\frac{\dot{Q}_w}{\Delta h_T} = \frac{\tau_w}{u_b} \, Pr^{-.67} \tag{9}$$

For no combustion between the flame and the wall $\Delta h_T = \Delta h$.
[9] into [5] with $\Delta h_T = \Delta h$ yields

$$C_H = \left. \frac{\tau_w Pr^{-.67}}{\rho_b u_b^2} \right\} \text{ flame to wall for heat transfer} \tag{10}$$

The skin friction coefficient is defined in the conventional manner by

$$\tau_w \equiv \frac{1}{2} \, C_f \rho_e u_e^2 \left. \right\} \begin{array}{l} \text{Boundary layer edge to wall for} \\ \text{momentum transfer} \end{array} \tag{11}$$

8

[11] into [10] yields

$$C_H = \frac{1}{2} C_f \frac{\rho_e u_e^2}{\rho_b u_b^2} Pr^{-.67}$$ [12]

[12] into [5] yields

$$\rho_f \dot{r} = \frac{1}{2} C_f \rho_e u_e B \, Pr^{-.67}$$

where

$$B \equiv \left. \frac{u_e}{u_b} \frac{\Delta h}{h_v} \right\} \quad \begin{array}{l} \text{a thermochemical parameter for a} \\ \text{particular propellant combination} \end{array}$$ [14]

It remains to find an expression for C_f in a turbulent boundary layer with mass addition at the wall.

For no blowing (66, p. 537)

$$\frac{C_{fo}}{2} = .03 \, Re_x^{-0.2}$$ [15]

Marxman (5) has developed an expression for C_f/C_{fo} which can be approximated by the following expression over the indicated range of the blowing parameter B'.

$$\frac{C_f}{C_{fo}} \approx 1.2(B')^{-0.77} \qquad (5 < B' < 100)$$ [16]

where

$$B' \equiv \frac{(\rho v)_w}{\rho_e u_e C_f/2}$$ [17]

Smoot, et al (C14) used Spalding's thin film model for the effect of blowing on wall friction. In that case

$$\frac{C_f}{C_{fo}} = \ln \left(\frac{1 + B'}{B'} \right)$$ [16a]

A comparison between Marxman's exact solution (5) and the approximate expressions [16] and [16a] is presented in reference C14 and indicates that [16a] is better than [16] for B' < 5 and that for B' from 10 to 25 they are equivalent. Most hybrids have been found to have a value of B' between 7 and 15.

[15] and [16] into [13],

$$\rho_f \dot{r} = .036 \ G \ Re_x^{-0.2} B \ Pr^{-0.67} (B')^{-0.77} \qquad [18]$$

where

$$G \equiv \rho_e u_e = \text{total mass flux of gas per unit area} \qquad [19]$$

and

$$Re_x = \frac{\rho_e u_e X}{\mu_e} = \frac{GX}{\mu_e} \qquad [20]$$

However, from [13] and [17] with $\rho_f \dot{r} = (\rho v)_w$

$$B' = B \ Pr^{-0.67} \qquad [21]$$

Thus with $Pr = 1$ [18] can be written

$$\rho_f \dot{r} = .036 \ G \ Re_x^{-0.2} B^{0.23} \qquad [22]$$

$$= 0.36 \ G^{0.8} \left(\frac{X}{\mu_e}\right)^{-0.2} B^{0.23} \qquad [23]$$

Equations [22] and [23] apply to incompressible flow and neglect variable fluid properties in the boundary layer. Marxman (5) has accounted for variable fluid properties in a semi-empirical manner and obtains

$$\rho_f \dot{r} = .036 \left(\frac{\bar{\rho}}{\rho_e}\right)^{0.6} \left(\frac{X}{\mu_e}\right)^{-0.2} G^{0.8} B^{0.23} \qquad [24]$$

where

$\bar{\rho}$ is a reference density and $(\bar{\rho}/\rho_e)^{0.6}$ generally has a value between 1.0 and 1.1 but is difficult to determine accurately.

To this point the following major assumptions have been made:
(a) flat plate turbulent boundary layer analysis is applicable.
(b) regression rate of grain is controlled by heat transfer from a diffusion flame.
(c) flame zone is infinitely thin.
(d) oxidizer enters port as a uniform gaseous stream.
(e) no heat transfer into subsurface region of solid grain.

(f)　Reynolds analogy is applicable between the flame and the wall in the presence of blowing.

(g)　Le = Pr = 1

(h)　Pressure is constant.

(i)　no radiation heat transfer to the wall.

It has been analytically determined (8) that the axial pressure gradient, which results from the mass addition (with no particle lag) induced flow acceleration, increases $\dot{Q}_c$ by approximately 10%. It was also shown (8) that particle lag losses (metalized grains, etc.) and the pressure gradient have only a negligible effect on the shear stress distribution and the velocity profile. Thus, the flat plate analysis is applicable with little error.

The blocking effect which reduces C_f normally depends only upon the gas mass addition at the wall since solids which leave the surface usually occupy negligible volume compared to the gas. If the hybrid grain has solids which do not vaporize, equation [1] should be written as follows (8)

$$\frac{Q_w}{\dot{r}} = \rho_f h_v = \rho_v h_{v_{eff}} = \rho_v \left(h_{v_b} + C_m \Delta T \frac{K}{1 - K} \right) \qquad [25]$$

where

$\Delta T = T_s - T_o$ = surface temperature – temperature deep in the grain

C_m = specific heat of nonvaporizing component

$\rho_v = \dfrac{m_v}{V} = \dfrac{\text{mass of vaporizing component}}{\text{volume of grain}} = \begin{array}{l}\text{bulk density of}\\ \text{vaporizing component}\end{array}$

$K = \dfrac{m_p}{m_f} = \dfrac{\text{mass of solid additives}}{\text{total mass of grain}}$

Thus, the binder is heated from temperature T_o and vaporized and the solid additives are heated from T_o to T_s. Equation [24] should now be written as follows

$$\rho_v \dot{r} = \rho_f (1 - K)\dot{r} = \frac{\dot{Q}_w}{h_{v_{eff}}} = .036 \left(\frac{\rho}{\rho_e} \right)^{0.6} \left(\frac{x}{\mu_e} \right)^{-0.2} G^{0.8} B^{0.23} \qquad [26]$$

11

where

$$B \equiv \frac{u_e}{u_b} \frac{\Delta h}{h_{v_{eff}}}$$ [27]

For hybrid systems which satisfy the above assumptions, equation [26] provides a working equation for internal ballistics calculations. However, it remains to be shown that B, given by equation [27], can be evaluated. Various combustion models have been employed in order to evaluate B. The various combustion models (5, 43, 47, 54, etc.) have been discussed by Marksman and Wooldrige (4). Marksman (5) assumed Le = Pr = 1 (similar profiles of enthalpy, concentration, and velocity) and used a Prandtl mixing length approach to solve for B, Ω, ϕ, and η_b

where

$$\Omega = \frac{K_{f_s} \nu}{K_{ox_e}}$$

ν = ox-fuel mass ratio

K_{f_s} = fuel mass fraction at surface

K_{ox_e} = oxidizer mass fraction at boundary layer edge

ϕ = velocity ratio = $\frac{u}{u_e}$

$\eta_b = \frac{y_b}{\delta}$ = dimensionless flame height

The complete solution (5) reveals that for all practical purposes B does not vary with axial distance, X (assuming that L/D < 25 so that severe oxidizer diluation does not occur). The mixture ration, ν, and the oxidizer mass fraction at the boundary layer edge, K_{ox_e}, were shown to decrease with increasing X. The concept of a nonstoichiometric diffusion flame has caused considerable debate (see discussion by Green in reference 4) and points out the present lack of understanding of transport processes in gaseous combustion.

12

3.3 Radiative Heat Transfer (3, 5, 7, and 8)

Radiation to the surface of the hybrid grain is treated as a grey-body equilibrium problem in which the flame zone (i.e., the gas or dense particle cloud) is considered as a radiating continuum. It has been pointed out (7) that this simplified approximation, although presently found to be adequate, may be inadequate in future systems which use more advanced propellant systems with high radiation heat transfer. However, improvements in this area will require a better understanding of nonequilibrium radiation than is presently available. The radiant heat flux is written

$$\dot{Q}_r = \sigma \epsilon_w \left(\epsilon_g T_r^4 - T_w^4 \right) \qquad [28]$$

$$\approx \sigma \epsilon_w \epsilon_g T_r^4 \qquad [29]$$

Now let

$$\epsilon_g = 1 - e^{-\alpha N z} \qquad [30]$$

where

T_r = effective radiation temperature

α = empirical constant

z = radiation path length

N = p for gas systems, or

= n, the number density of particles, if there are radiating solids in the gas

Then,

$$\dot{Q}_r = \sigma \epsilon_w T_r^4 \left(1 - e^{-\alpha N z} \right) \qquad [31]$$

Reference 17 presents a derivation of an expression for ϵ_g which is considerably more complex than equation [30]. Although the derivation yields a better understanding of the parameters affecting ϵ_g, the resulting expression does not appear to offer any distinct advantage over equation [30].

3.4 Combined Convective and Radiative Heat Transfer

The regression rates of most metalized hybrid grains will result from both convective and radiative heat transfer. The effects of $\dot{Q}_r$ and $\dot{Q}_c$ cannot be summed directly since $\dot{Q}_c$ is a function of the surface mass addition.

Increased heat flux to the wall increases $\dot{r}$ which increases the surface blowing. This, in turn, decreases $\dot{Q}_c$ but has little effect upon $\dot{Q}_r$. Neglecting $\left(\rho/\rho_e\right)^{0.6}$ and letting $Pr = 1$, equations [18] and [31] yield.

$$\rho_v \dot{r} = \frac{\dot{Q}_w}{h_{v_{eff}}} = .036 \; G \; Re_x^{-0.2} \left(B_r'\right)^{-0.77} B + \frac{\sigma \varepsilon_w T_r^4 \left(1 - e^{-\alpha Nz}\right)}{h_{v_{eff}}} \qquad [32]$$

The thermochemical parameter B is not significantly effected by $\dot{Q}_r$ but the blowing parameter B' will increase with increasing $\dot{Q}_r$.

$$B_r' = B' \text{ with radiation} = \frac{(\rho v)_w}{\rho_e u_e C_{fr}/2} = \frac{\rho_v \dot{r}}{\rho_e u_e C_{fr}/2} \qquad [33]$$

[33] into [32] yields

$$h_{v_{eff}} B_r' \rho_e u_e C_{fr}/2 = h_{v_{eff}} .036 \; G \; Re_x^{-0.2} \left(B_r'\right)^{-0.77} B + \dot{Q}_r \qquad [34]$$

From [13]

$$\dot{Q}_c \big|\text{without radiation} = \left(C_f/2\right) \rho_e u_e B \; h_{v_{eff}} = .036 \; G \; Re_x^{-0.2} B^{0.23} \; h_{v_{eff}} \qquad [35]$$

Dividing [34] by [35] and noting that

$$\frac{C_{fr}}{C_f} = \left(\frac{B_r'}{B}\right)^{-.77} \qquad [36]$$

yields

$$\frac{B_r'}{B} = 1 + \frac{Q_r}{Q_c} \left(\frac{B_r'}{B}\right)^{0.77} \qquad [37]$$

[37] may be approximated by (3)

$$\frac{B_r'}{B} \approx e^{1.3 \frac{\dot{Q}_r}{\dot{Q}_c}} \qquad [38]$$

From [32]

$$\rho_v \dot{r} = \frac{\dot{Q}_c}{h_{v_{eff}}} \left(\frac{B_r'}{B}\right)^{-0.77} + \frac{\dot{Q}_r}{h_{v_{eff}}} \qquad [39]$$

14

[38] into [39] yields

$$\rho_v \dot{r} = \frac{\dot{Q}_c}{h_{v_{eff}}} \left(e^{-\dot{Q}_r/\dot{Q}_c} + \frac{\dot{Q}_r}{\dot{Q}_c} \right)$$ [40]

where $\dot{Q}_c$ is the convective heat transfer without radiation.

Equation [40] indicates the coupling that occurs between the convective and radiative heat transfer. Two systems may have the same regression rate but if $\dot{Q}_r$ is significantly present in one, then it will respond differently to changes in pressure, oxidizer mass flux, and etc. From equation [31] it is observed that the radiation heat transfer is not directly dependent upon G as is the convective heat transfer. However, increased G (which increases $\dot{r}$) does affect $\dot{Q}_r$ by increasing the pressure and/or the number density of radiating particles (C6). Reference C14 (which used the Spalding thin film model) arrives at a somewhat different result than equation [37]. The $\dot{Q}_r - \dot{Q}_c$ coupling is handled by a graphic technique (without making an approxi-mation corresponding to equation [38]) and the results indicated that equa-tion [40] underestimates $\dot{r}$ for $\dot{Q}_r - \dot{Q}_c$ coupled systems with B < 100. However, equation [40] presently appears to be adequate and more readily lends itself to an internal ballistics model which can be programmed for engineering design.

In summary

$$\rho_v \dot{r} = \rho_f (1 - K) \dot{r} = \frac{\dot{Q}_c}{h_{v_{eff}}} \left(e^{-\dot{Q}_r/\dot{Q}_c} + \frac{\dot{Q}_r}{\dot{Q}_c} \right)$$ [41]

$$\dot{Q}_c = .036 \ h_{v_{eff}} \left(\frac{\bar{\rho}}{\rho_e} \right)^{0.6} \left(\frac{X}{\mu_e} \right)^{-0.2} G^{0.8} B^{0.23}$$ [42]

$$\dot{Q}_r = \sigma \epsilon_w T_r^4 \left(1 - e^{-\alpha N z} \right)$$ [43]

$$G = \rho_e u_e = \left(\frac{\dot{m}_g}{A_p} \right)$$ [44]

These equations, when coupled with grain geometry and theoretical performance data provide the necessary material for an internal ballistics computer pro-gram which can be used for engineering design and analysis.

3.5 Parameter Evaluation

In order to use equations [41] through [44] for internal ballistics calculations several parameters must first be evaluated. Inspection of the equations indicates that the following parameters must be evaluated: $h_{v_{eff}}$, h_{v_b}, T_s, T_r, B, Δh, $(O/F)_{flame}$, $(\bar{\rho}/\rho_e)$, ε_w, α, A_p, $\dot{m}_g$, n, T_e, and z.

Details of the approximations and/or calculations used in order to obtain these parameters can be found in the literature (8, 12, C4, etc.) and will only be briefly considered.

$h_{v_{eff}}$ – the effective heat of gasification of the solid phase. This parameter must be calculated or determined experimentally. In order to determine $h_{v_{eff}}$ analytically, equation [25] indicates that the surface temperature, T_s, the effective heat of vaporization of the binder, h_{v_b}, and the specific heat of the nonvaporizing solids, C_m, must be evaluated. The latter parameter is normally known or can be estimated from the literature.

h_{v_b} – the effective heat of vaporization of the binder. The thermal decomposition processes of the binder must be understood if h_{v_b} is to be analytically determined. Calculation of h_{v_b} for simple polymers such as plexiglass have been made (12, etc.). For more complex binder systems or solid grains which incorporate solid oxidizers, h_{v_b} will normally have to be evaluated experimentally. h_{v_b} may be significantly reduced due to surface reactions caused by the oxidizer in the grain. In addition, for those systems in which oxidative attack of the binder is important (discussed below), one method of accounting for this pressure sensitive effect would be through a pressure sensitive h_{v_b} (or $h_{v_{eff}}$).

T_s – the surface temperature. T_s is normally estimated to be between 600 and 800°K (39).

16

T_r - the effective radiation temperature. This parameter is estimated with any error absorbed in the empirical radiation parameter α. UTC (8, 12, etc.) has used $T_r = 2/3$ ($T_{stoichiometric}$). In radiating systems, the effect of the local mixture ratio, O/F, can be significant. As noted above, O/F is found to decrease with increasing distance from the head-end. Any initial dilution of the oxidizer (N_2 in air, etc.) will also significantly reduce T_r. A reduction of O/F also occurs when a hybrid motor is throttled and this will affect T_r. These effects can be readly included in the current model by treating T_r as T_r (x,t) or T_r (O/F).

B - the mass transfer number. Equation [27] indicates that evaluation of B requires evaluation of u_e/u_b, Δh, h_{veff}. As pointed out in references 8, 12, etc., even significant errors in the determination of Δh and h_{veff} do not significantly effect the calculated $\dot{r}$ in convective heat transfer limited systems since in those systems $\dot{r} \propto B^{0.23}$. In order to find u_e/u_b (5, 17) it is necessary to evaluate (ρ/ρ_e), which may be determined analytically (5). In UTC's current internal ballistics program (12), .036 $(\rho/\rho)^{0.6}$ in equation [42] is treated as an empirical constant which is determined experimentally.

O/F - the local mixture ratio at the flame. The value of O/F at the flame is known to be fuel rich (3, 32). UTC (8, 12, etc.) estimates O/F to be (3/4) (O/F)$_{stoichiometric}$ and considers it to be independent of X. However, as noted above, O/F may vary with axial position, oxidizer, dilution, and throttling. Although this does not normally affect $\dot{Q}_c$ significantly it may affect $\dot{Q}_r$ (i.e., T_r) as indicated above. It appears that a variable O/F could be incorporated into the current model without too much difficulty.

Δh – the sensible enthalpy difference between the flame and wall. Once the local mixture ratio and the surface temperature are known or estimated, Δh can be readily calculated (10).

ε_w – the effective wall emissivity. ε_w is estimated. Muzzy (12) has used $\varepsilon_w = 0.90$.

α – the empirical radiation parameter. α is determined from experimental data.

A_p – the effective port area (that yields an average u_e). A_p is the actual port area less the area occupied by the boundary layer displacement thickness (12).

$\dot{m}_g$ – the total mass flux of gas in the port. $\dot{m}_g$ varies with the axial position, X. It must account for the mass flux of gas entering the port, the mass flux from the decomposing solid grain, and any gas consumed in the production or comsumption of solid particles (10). In addition, it should be emphasized that $\dot{m}_g$ includes only the vaporized oxidizer (C10). The model assumes uniformly injected vapor. However, in real systems the time required for complete vaporization of the oxidizer may be significant. This correction could be included in the current model but depends upon knowledge of the initial drop size distribution which is not readily determined.

n – the number density of particles in the flame zone. The contribution of the radiation from the gas is neglected when radiating particles exist in the flame zone. Expressions have been developed for calculating n (8, etc.) depending upon the source of the particles. The resulting expressions show that n varies as $p/\dot{m}_g$ and is a function of the axial position, X. In the evaluation of n, the gas temperature (T_e) at the boundary layer edge, must be determined. An approximate method for evaluating T_e has been presented (8, 10, 12, etc.).

z - the optical path length. This parameter is not well under-
stood since actual systems have nonequilibrium radiation.
Reference 8 has considered z proportional to the hydraulic
diameter of the port for radiating gaseous systems and as
a constant (proportional to flame height) for radiating
particle systems. Any inaccuracies in n or z are accounted
for in the empirical radiation parameter α.

μ_e, Pr,

C_p, etc. - these parameters are calculated by normal techniques, or
estimated.

As indicated above, the regression rate model must be coupled with
grain geometry equations and performance parameters such as the ideal thrust
coefficient, C_F, the ideal characteristic exhaust velocity, C*, and the com-
bustion and nozzle efficiencies. These parameters are evaluated in the same
manner as for liquid or solid propellant systems. Several grain configura-
tions and propellant combinations have been evaluated. References C4 and
C5 present calculated and/or measured values of B, Δh, etc., for various
propellants. In addition, the literature contains many references which
present theoretical performance calculations for various hybrid propellant
combinations (22, etc.).

3.6 Typical Theoretical Results of the $\dot{Q}_r - \dot{Q}_c$ Coupled Hybrid Model

Figure 2, from reference 12, indicates the type of results which are
obtained from the $\dot{Q}_r - \dot{Q}_c$ coupled model for a typical aluminized grain. At
low values of G_o, radiation heat transfer dominates the total heat flux to
the surface and therefore, the regression rate becomes practically inde-
pendent of mass flux through the port but very sensitive to changes in
operating pressure and mixture ratio. At high values of G_o, convective heat
transfer dominates and the regression rate becomes insensitive to pressure
changes but varies as $G^{0.8}$. When the mass flux is at some intermediate
value, both $\dot{Q}_c$ and $\dot{Q}_r$ are significant and the regression rate depends upon
pressure, mass flux, port diameter, and grain length. The slight increase
in $\dot{r}$ at low G_o predicted for radiating systems as shown in Figure 2 results

19

from the increase in n ($n \propto 1/\dot{m}_g$), and therefore $\dot{Q}_r$, at low G_o (10). However, the model assumes T_r is independent of G_o and, if T_r is allowed to vary as discussed above, the form of the curve at low G_o may change. Although some experimental data exist (40, 41) for $\dot{Q}_r$ at low G_o, much additional experimental and analytical work is required.

3.7 Other Heat Transfer Limited Models

As mentioned above, the heat transfer limited hybrid internal ballistics model has been developed into a practical design tool by both the United Technology Center (UTC) and the Lockheed Propulsion Company (LPC). The discussion above dealt primarily with the model as used by UTC. The LPC work (C13, C14, C15) is very similar. Although LPC handled $\dot{Q}_r - \dot{Q}_c$ coupling, $C_f = C_f(B')$, and condensed surface species in a somewhat different manner, practical application of either treatment remains essentially the same.

Other investigators have used somewhat more simplified analyses for heat transfer limited hybrid operation (50, 51, etc.). Recently Drew (52) has used the basic UTC model for systems in which a metallic melt layer is formed on the surface of the grain. In the analysis it is assumed that the melt layer does not affect the gas-phase heat transfer. Lieberherr (26) has also considered the effects of a metallic melt layer on the surface.

Various investigators from Japan have also studied hybrid combustion (59, 74). Reference 59 used a Rayleigh flow model in place of the boundary layer treatments discussed above. These authors treat the flow as laminar which is normally not applicable to hybrid environments.
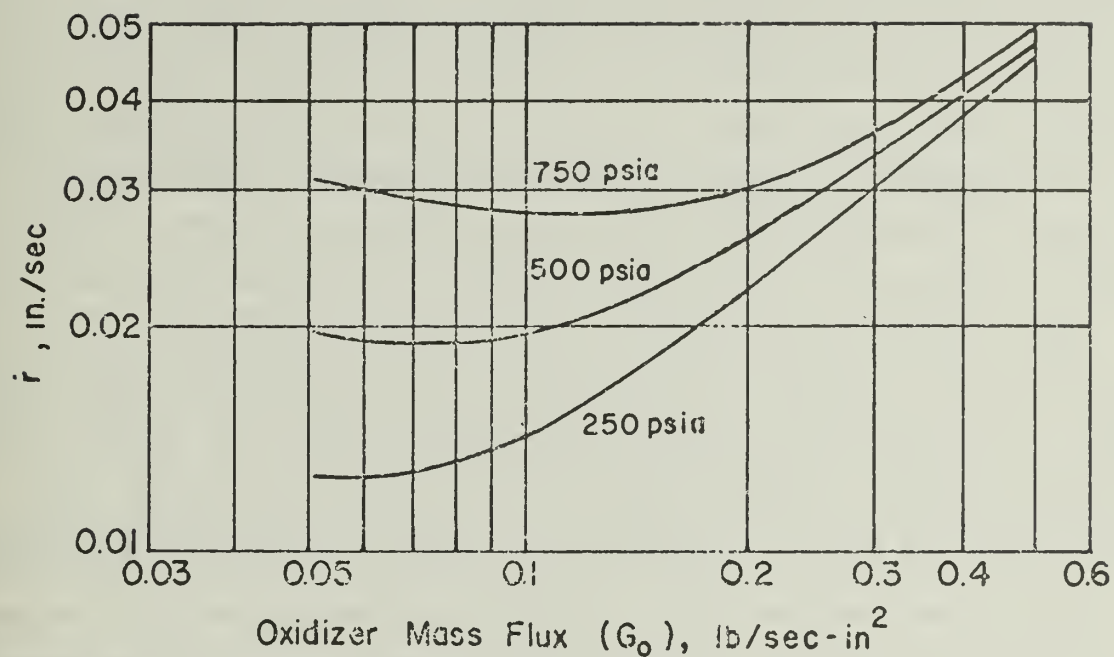
Fig.2 REGRESSION RATE DEPENDENCE ON OXIDIZER MASS
FLUX AND PRESSURE FOR A TYPICAL ALUMINIZED
HYBRID SYSTEM (from p.51 Ref.12)

4.0 KINETIC EFFECTS IN HYBRID COMBUSTION

4.1 Introduction

As mention above, the heat transfer limited model has been found to be adequate for many propellant systems over restricted mass flux and pressure regimes. However, experimental data at low pressure and high mass flux (8, 17, 27, 28, 29, etc) indicate that the heat transfer limited model is not applicable in these regimes for many propellant systems. Smoot and Price (29) have found that at high G, low P, the regression rate becomes independent of G but is directly related to the partial pressure of the oxidizer in the gas phase. In addition they have found experimentally (27, C14) that the addition of metal to the solid grain reduced the pressure dependence of $\dot{r}$.

There has been considerable debate concerning which kinetic mechanism(s) in the overall combustion process become rate controlling in these limiting cases. The various mechanisms will be discussed below but first it is important to briefly consider whether or not kinetic effects need to be studied from the practical standpoint of expected operational environments.

It is true that for a wide range of propellant systems that the regression rate is dominated by heat transfer over much of the expected range of mass flux. In these regimes of operation the heat transfer limited model can be expected to be adequate for internal rates and minimum grain surface area the trend in some future hybrid systems will undoubtably be towards the highest mass flux possible. The practical upper limit on G_o or G depends on ignition and flooding effects which will be discussed below. However, at these high mass fluxes, kinetic effects may become significant. The propellant system could still be characterized by sub-scale testing and/or the heat transfer model could be modified to include an empirical correction term which is pressure dependent (10), but neither of these solutions provide any understanding of the basic phenomenon. This understanding will be required if hybrid systems of the near future are to be adequately modeled and scaled.

There are several other developments which require an understanding of the kinetic effects in hybrid combustion. Solid oxidizers are being included in the fuel grain in order to obtain higher regression rates and

smaller variation in I_{sp} with O/F. As the percent of oxidizer is increased, the hybrid system will become more like a conventional composite solid propellant in which kinetic mechanisms dominate. A somewhat common limiting problem exists for hybrid systems with significant oxidizer in the solid grain, as well as for solid propellants subjected to high gas velocities parallel to the surface – i.e., in erosive burning environments. An understanding of the interrelationships between the kinetic mechanisms and the turbulent transport mechanisms would be of great value in both of these closely related areas.

There is currently a considerable interest in air-breathing hybrids and solid-fuel ramjets. The heat transfer model discussed above can be expected to provide the basic model for these systems since oxidizer dilution has been considered in some detail for hybrids (assuming the grains are not kinetically dominated and if the boundary layer flow is not significantly altered). When the oxidizer is diluted the flame moves further away from the surface, and increases in thickness (3), while the flame temperature drops (C4). An analysis by UTC (C4) has shown that the primary effect of dilution in $\dot{Q}_c$ dominated systems is on the parameter ΔH (and therefore B). Dilution may also significantly effect $\dot{r}$ in a $\dot{Q}_r$ dominated system through the parameter T_r.

Significant kinetic and radiation effects can be expected in many air-breathing applications and these aspects have received only limited attention. A recent analysis (discussed below) has indicated that surface catalytic attack by active oxidizing species at the grain surface may be the controlling mechanism in the high G, pressure dependent regime of hybrid operation. If this proves to be true, then it may be found that as the oxidizer is significantly diluted, pressure sensitive combustion may occur at significantly lower mass fluxes than found in conventional hybrids. Variable thrust hybrids, in which the oxidizer is throttled, may also present an operational environment in which the combustion mechanism is controlled by heat transfer at high thrust (high pressure) and controlled by kinetics at low thrust (low pressure).

High frequency combustion instability has been observed only rarely in conventional hybrids. Combustion instability will be discussed below but is mentioned here because of its obvious link to kinetically controlled combustion.

When kinetic mechanisms contribute significantly to the hybrid combustion process (air-augmented systems, high oxidizer content in solid grain, low pressure from throttling, etc.) combustion instability may become a significant problem.

It is apparent that the kinetic aspects of hybrid combustion may be of considerable importance in the near future and an understanding of these phenomena is required. As mentioned above, there has been considerable debate in the literature concerning whether the kinetically influenced regime of combustion occurs in the gas phase at the flame or heterogeneously at the wall. A brief review of the literature related to the kinetic aspects of hybrid combustion is presented below.

4.2 Kinetic Modeling

Until recently most investigators have considered that the pressure sensitive regime of hybrid combustion was due to either gas phase kinetics or exothermic heterogeneous reactions at the surface. In addition, several investigators have studied hybrid regression rates by considering solid phase kinetics.

Smoot and Price (27, 28, 29, C13) found experimentally that the regression rate of LiH/rubber fuels was a function of G_o and the partial pressure of active oxidizer but practically independent of total pressure. Their experimental data also indicated that burning rate catalysts had no effect on pressure sensitivity and that the addition of metal reduced the pressure sensitivity. They reasoned from this data that the pressure sensitivity of $\dot{r}$ was controlled by the turbulent transport of active oxidizer to the wall and exothermic first order heterogeneous reaction of this active oxidizer with the solid fuel. Solid fuel decomposition, radiative heat transfer, and binary gas phase reactions were not considered to play a dominant role. An analytical development is presented in reference C13. Houser and Peck (31) have also indicated that oxidative degradation is required to explain the high regression rates of hybrid fuels. In contrast Miller (42) considered the pressure sensitivity to result from reduced reaction rates at low pressures which broaden the gas phase flame zone.

24

Kosdon and Williams (33) have attempted to explain $r = \dot{r}(p)$ by considering small perturbations to the heat-transfer limited regime. Marxman (8, 17) has discussed the limitations of this model. An interesting approach has been presented by Tsuge and Fujiwara (59) who have argued that a Rayleigh flow model better fits the hybrid situation of high rates of wall mass addition than the boundary layer model. Their treatment was restricted to laminar flow and kinetic behavior was introduced through an Arrhenius expression for surface pyrolysis. The model correctly indicated the independence of $\dot{r}$ on G at high values of G.

Rabinovitch (48) studied the thermal degradation of polymers. His study indicated that the wall temperatures found in hybrid rockets are adequate to produce the observed hybrid regression rates through thermal degradation alone. Recently Lengelle (40) has also studied thermal degradation kinetics but the results were inconclusive. Marksman (8, 8, 17) has also analyzed pressure sensitive hybrid combustion and, supported by Rabinovitch's work, has argued that gas phase kinetics in the flame zone are the principal controlling mechanism during low pressure, high G_o operation. His development (7) indicated that the pressure sensitive regression rate cannot be explained solely by heterogeneous surface kinetics. Subsequently, Marxman, et al (17) have conducted experiments in the pressure sensitive regime. Two binders (PU, PBAN), three oxidizers (O_2, F_2, N_2O_4), and various aluminum loadings (0, 20, 40%) were employed. Tests were conducted with varying G_o and varying throat area. Schlieren studies also showed that the flame zone broadens at low pressures. The experimental results indicated that the percent aluminum and the binder decomposition did not significantly affect the pressure dependence of $\dot{r}$ (although considerable data scatter existed at the lowest pressures). The former result for aluminum effects is in contrast to the results of Smoot and Price for LiH (28). Lieberherr's (26) work suggests that the pressure sensitivity of hybrid grains containing metallic hydrides may be due in part to surface melting and liquid flow.

Marksman, et al (17) reasoned that "at high pressures the gas-phase reaction rates are fast enough to consume all of the available material that is being vaporized at the surface." It is further reasoned that gas-phase reaction rates decrease with decreasing pressure until the flame cannot

consume all vaporizing material. This results in a shift to a mixture ratio which is even more fuel rich, and thus $\dot{Q}_w$ is reduced. The reduction in $\dot{Q}_w$ decreases $\dot{r}$. An analytical expression was derived for the pressure sensitivity of the steady-state regression rate. The turbulent diffusion flame which exists at high pressures was considered to broaden and transition to a turbulent pre-mixed flame at low pressures. By using wrinkled flame theory (64) for the premixed turbulent flame and Denison and Baum's (63) result for the kinetic flame speed, an expression is developed for $\dot{r}$ in the pressure sensitive regime. The resulting expression adequately fit the experimental data for the pressure sensitivity of $\dot{r}$ at a fixed G_o but did not properly predict the independence of $\dot{r}$ on G at high G. Additional experimental and analytical work is needed to determine the validity of this model.

Recently Stickler and Kumar (38, 39) have studied the thermal degradation of polymers. Primary consideration was for polymethyl methacrylate (PMM). They have shown that thermal degradation cannot by itself account for the high regression rates of hybrids with wall temperatures within the range reported in the literature (600-800°K). It is argued that gas phase kinetics are not slow enough and that exothermic heterogeneous reactions require more oxidizer at the wall than reported experimentally. An analysis of turbulent transport through the flame zone is presented and the conclusion is reached that the small percent of active oxidizer that diffuses through the flame to the wall acts as a catalyst for depolymerization. Thermal degradation reduces the polymer to large fragments and the catalytic action is considered to break down the fragments to a size which has a vapor pressure at the surface equal to the chamber pressure. The rate of catalytic depolymerization is considered proportional to the oxidizer concentration at the wall. It is argued that at high pressures the catalytic rate is sufficiently high that the burning process becomes heat-transfer (thermal degradation) limited. At lower pressures and high G, the regression rate is limited by oxidizer transport to the wall. In this limit there is not enough oxidizer at the wall to adequately degrade the polymer for vaporization and $\dot{r}$ becomes practically independent of G. The authors have pointed out that the kinetics of oxidative degradation are not well understood and therefore, it will be difficult to actually predict regression rates in the pressure sensitive regime.

The work of Stickler and Kumar appears to have merit although the observed broadening of the flame zone at low pressures (17), the lack of effect of various binders on the pressure sensitivity (17, 27), and the possible pressure sensitivity of the formation and consumption of metallic melt layers (26) indicate that pressure sensitive hybrid combustion is a complex process which probably involves more than one pressure sensitive mechanism.

The above discussion indicates that a considerable lack of understanding still exists in the area of pressure sensitive hybrid combustion. Although several new analytical models have appeared in the past few years, very few basic experimental studies have been conducted. Additional experimental investigations are required. The inherent data scatter obtained at low pressures needs to be considered further. A real need exists for an experimental method that allows accurate determination of the regression rate during combustion.

In addition, the similarity between the treatments for the rates of thermal degradation of polymers (38, 30, 48, 40, etc.) and the rates of fracture of polymers subjected to compressive or tensile loads (67, 68, 69, 70, etc.) requires investigation.

5.0 APPLICATION, IMPLICATIONS, AND LIMITATIONS OF
THE HYBRID INTERNAL BALLISTICS MODEL

5.1 General Discussion

Although the current internal ballistics model does not include kinetic effects, it can be used for many propellant combinations over a fairly wide range of mass flux. There are many important design questions which can be answered by utilizing the heat transfer limited model. In addition, there are areas of hybrid operation which are outside the basic model assumptions.

It is not the purpose of this section to discuss in detail all of the applications and limitations of the current model which have been considered, but rather to indicate some of the major considerations and to reference the literature where pertinent discussions have been presented. The discussion below is arbitrarily divided into three sections: Ignition and Mass Flux Considerations, Regression Rate Tailoring and Injector Effects, and Performance. These topics are obviously interrelated.

5.2 Ignition and Mass Flux Considerations

With the natural trend toward the use of high oxidizer mass flux, it becomes necessary to better understand the internal ballistics in this "limiting" regime of operation. Each propellant combination has an upper and lower mass flux beyond which operation becomes irregular and the ballistics model is not applicable.

As discussed above, for high mass fluxes many propellant systems become dominated by kinetic effects. However, even those systems that remain insensitive to pressure into the high mass flux range can reach flow rates in which "Flooding" occurs. Ignition becomes a problem at high G_o and the burning process may exhibit periodic pressure spikes or extinguish completely. Wooldridge and Muzzy (10) have discussed come of the problems related to ignition and operation at high mass fluxes. In the case of hypergolic propellant combinations, high G_o operation can cause excess oxidizer to accumulate over a portion of the grain which may subsequently ignite and cause severe pressure spikes. In non-hypergolic propellant applications the high G_o may prevent adequate oxidizer vaporization after ignition and combustion may be extinguished.

Various methods of hybrid ignition have been considered by Muzzy, et al (C10). Hypergolic propellant combinations, solid propellant pyrotechnics, solid propellant ignition boosters to enhance ignition (C9) (which degrade the safety advantage of hybrids), and hypergolic liquid bipropellants have been used.

Ignition lags from several hundred milliseconds down to tens of microseconds have been observed (C7, 22). Various techniques have been employed to decrease the ignition lag (C8, etc.). In general, it is advantageous to minimize ignition lags in order to obtain higher reliability and repeatibility and to prevent ignition pressure spikes (22). Ignition overpressure is highly undersirable. In addition to the degradation or destruction of structural integrity, these overpressures can cause an initially high mixture ratio (O/F) which may result in significant throat erosion in the first few seconds of operation (C9). This throat erosion in turn can significantly degrade the overall performance and decreases the utility of the internal ballistics computer program.

Very few studies of ignition phenomena in hybrids have appeared in the literature. Barrere (22, 25) has initiated work in this area and other investigations are also pertinent (34, 75, C12, etc.). The ignition process, both initially and in restart operations, is probably one of the least understood phenomena in hybrid combustion. Many design variables appear to effect the ignition process. Currently many questions remain partially or fully unanswered. For example, what is the best way to distribute the ignition energy and/or the oxidizer flow over the hybrid grain (uniformly, head end, in valley, on plateaus, etc.)? The ignition energy can also be distributed in various forms--gases, solid-liquid interaction, etc. What is the optimum amount of ignition energy consistent with the propellant formulation, G_{ox}, etc? Can the hybrid tolerate overignition? What actually causes the ignition? What effect does initial oxidizer temperature have on ignition (C4)? What is the optimum time between ignition and oxidizer flow initiation? Will addition of solid oxidizer to the solid grain insure greater ignition reliability and less ignition lag? These are some of the problems which must be considered. There is an obvious need for basic studies in ignition which would hopefully lead to at least a qualitative model for the hybrid ignition process.

At the other extreme of mass flux-low G_o-a "limiting" G_o also exists, but for considerably different reasons than at high G_o. Problems associated with low mass flux and the resulting low regression rates of the solid grain appear when variable thrust operation is considered. Boost-sustain duty cycles or continuous throttle capability often necessitate very low G_o, low $\dot{r}$ operation. In addition, when restart capability is required, heat soak into the solid grain between firings must be considered. Ignition at low G_o may also be expected to produce undersirably long ignition delays.

At very low G_o several model assumptions may be violated. The turbulent boundary layer may transition to laminar and this may explain the significant data scatter typically found at low G_o (C14). In addition, heat may penetrate to significant depths below the surface resulting in a nonsteady regression rate.

Significant pressure effects may also be present at low G_o in metalized systems due to the pressure sensitivity of the radiative heat transfer.

The problems associated with transient operation (ignition, restart, throttling) and operation at low G_o, low $\dot{r}$ have received some analytical attention by several investigators (3, 7, 8, 10, 50, etc.). After ignition or throttling, a period of time is required to establish the steady-state burning rate (temperature profile in the solid grain). This time may vary in restart systems depending on the soak period between firings (50). It has been shown (8, 10) that this time is critical for solid grains which have a high vaporization temperature or a low enthalpy of vaporization. Operation at low $\dot{r}$ and/or in the transient regime may cause subsurface material to be chemically or physically altered (charred, melted, decomposed) which in turn can effect both steady-state combustion and restart capability (C16). Operation in this regime may also result in low frequency pressure oscillations associated with alternate accumulation and removal of the melted or charred layer on the surface (17).

Current practice is to run subscale tests to determine the upper and lower mass flux limits for a particular propellant combination. Incorporating transient phenomena directly into the current internal ballistics program would be difficult. Subscale data and results from simplified transient analyses can be used as "corrections" to the ballistics model during the transient portion of the duty cycle.

The internal ballistics of throttled hybrids within the heat transfer limited operating regime requires additional considerations beyond those for throttled liquid systems. This occurs because of the coupling between $\dot{r}$ and G or G_o. A brief consideration of the implications of this coupling is presented below. Details can be found in references 10, C4, C6, C10, and etc.

Various methods of obtaining thrust-time (F - t) variations in hybrids have been discussed in the literature (10, C4, C10, etc.). Programmed F - t missions can normally be handled through appropriate grain design; geometry, multiple castings, etc. When demand F - t is required, some form of oxidizer throttling is normally required. Throttling of head-end oxidizer is the simplest method (provided adequate performance and stability can be obtained without aft injection) but has a distinct disadvantage. When G_o is reduced G_f decreases, but only to a much lesser extent since r varies approximately as $G_o^{0.5}$. Thus, as the system is throttled the pressure drops, the O/F decreases, and in many cases this would result in decreased performance.

For demand thrust, reference C4 has discussed several alternatives: (1) provide excess propellant so that the motor can operate off optimum O/F, and sacrifice the performance; (2) use fore and aft-end oxidizer injection in which the combustion products would be fuel rich for all head-end G_o. Aft-end G_o would then be varied to obtain the desired thrust at the optimum mixture ratio (10); (3) use a propellant system with a pressure sensitive regression rate so that the fuel flow rate varies more directly with the oxidizer flow during throttling. However, operation in this pressure sensitive regime may introduce other difficulties such as combustion instability, etc.; (4) develop a propellant with a large value of n where n is the exponent in the regression rate expression $\dot{r} = aG_o^n$. High values of n would allow throttling at practically constant O/F but present problems with regard to matching grain progressivity with the resulting rapid decrease in $\dot{r}$ that would occur as the grain was consumed. Multiple port designs with separate oxidizer injectors have also been used (C10). Throttling is accomplished by shutting off some of the injectors.

The throttling capability of hybrids is one of its inherent advantages over conventional solids. It is difficult to estimate what throttling ratio can be achieved without undue loss of performance or complexity of design. This will depend on propellant choice and future techniques for throttling.

31

5.3   Regression Rate Tailoring and Injector Effects

Because of the interdependence of most of the operating parameters in
hybrid internal ballistics it is difficult to separate discussion of the
various aspects. Two important areas that are isolated in this section are
regression rate tailoring and the problems from injector effects, but the
discussions are keyed to interrelationships with maxx flux and ignition.

5.3.1 Regression Rate Tailoring. One of the problems with hybrid propul-
sion systems is the very low regression rate which characterizes most pro-
pellant combinations. Various techniques are needed for increasing the rate
($\dot{r}$) and for obtaining time variations. Good detailed discussions can be
found in references 8, 12, C4, C5, and C15.

From equations [27] and [42] it is observed that for convective heat
transfer limited systems

$$\rho_v \dot{r} = \rho_f (1 - K) \dot{r} \propto G^{0.8} \left(\frac{\Delta h}{h_{v_{eff}}} \right)^{0.23} X^{-0.2} \qquad [45]$$

For heat transfer limited systems in which both convective and radiative
effects are present the situation is more complex and equations [41] through
[44] must be considered simultaneously. In their applicable ranges these
equations indicate what can and cannot be done to effect a significant in-
crease in $\dot{r}$. In addition, they point to other techniques (with reservation)
which may be employed that are beyond the limitations of the model.

In $\dot{Q}_c$ - limited systems, nominal changes in Δh and $h_{v_{eff}}$ have little
effect upon regression rate since $\rho_v \dot{r} \propto \left(\Delta h / h_{v_{eff}} \right)^{.23}$. Therefore, addition
of exothermic material to the solid grain to reduce $h_{v_{eff}}$ cannot be expected
to significantly increase $\dot{r}$ (C4). However, by adding solid materials that
do not vaporize at the surface, but rather leave the surface as solids, ρ_v
is decreased and $\dot{r}$ increased correspondingly. There is an obvious practical
limit to the amount of solids which can be added. If adequate overall per-
formance is to be obtained solids must be efficiently burned in the gas
phase (8). It should also be noted that for the upper limit where all solids
are consumed in the gas phase, a significant amount of oxidizer may be con-
sumed above the flame, thereby reducing the partial pressure of the oxidizer.

In systems which are dominated by radiative heat transfer, $\dot{r}$ may be significantly effected by changes in Δh or $h_{v_{eff}}$ since surface blowing does not counter the heat transfer as readily as it does in $\dot{Q}_c$ systems.

The above discussion has indicated several means by which the regression rate can be increased; addition of highly exothermic material to the solid grain, addition of nonvaporizing solids to the solid grain, and increasing the radiative contribution to the overall heat trnasfer to the grain. The latter is normally accomplished by increasing the metal content of the grain which increases the effective radiation temperature, T_r. The maximum metal content is limited by the amount that can be efficiently consumed in the overall combustion process. In addition, high metal loading may cause a melt layer to form on the surface. As mentioned above, the metallic melt layer may be periodically blown off, giving rise to low frequency pressure oscillations or undesirable combustion characteristics (C16). The formation of the metal melt also requires some modification to the ballistics model as discussed by Drew (52) and Lieberherr (26).

Another method of increasing $\dot{r}$ is by the addition of solid oxidizer to the grain. The quantity must be limited to prevent self-sustaining combustion if hybrid operation is to be maintained. Reference C4 has indicated that this limiting quantity depends both on the type and size of the solid oxidizer crystals. Addition of solid oxidizer will also introduce kinetic effects and the regression rate will become pressure dependent with increased possibility of combustion instability. The gases leaving the surface and entering the diffusion flame probably consist of binder and oxidizer decomposition products and the products from fuel rich combustion near the surface. Consequently, the quantity of solid oxidizer that renders the current heat transfer ballistics model inadequate is likely to be considerably less than the maximum allowed for non-selfsustaining operation.

Reference C4 has also discussed the possibility of adding materials to the grain which yield gas-producing reactions below the surface. This would cause mechanical ejection of solid materials into the gas stream where they must then be consumed.

It has been found that port configurations that induce mixing produce higher average regression rates. Reference C4 reports that a triangular port produced a significant increase in $\dot{r}$ over the conventional circular port.

The addition of an orifice (diaphragm, restrictor) to the grain to induce large-scale turbulent mixing has been found to increase the regression rate downstream of the orifice by as much as 50% (17, 22, 24). Various additives have been found which significantly increase $\dot{r}$ at high pressures and produce pressure sensitive combustion (15).

It is interesting that the heat transfer limited model can point to methods by which $\dot{r}$ can be increased but at the same time it is not applicable in many of the resulting combustion situations.

5.3.2 <u>Injector Effects</u>. Injector effects are one of the more important aspects of hybrid combustor design. Injector and head-end design can significantly affect the oxidizer vaporization rate and oxidizer droplet penetration of the boundary layer. Since $\dot{r} \propto G^{0.8}$, injectors that gouge out the solid grain at the head end (due to wall impingement of liquid or vaporized oxidizer or to severe recirculation zones) may significantly alter the regression rate throughout the entire grain and alter the expected performance. These effects can easily overshadow all ballistic model calculations. Large injector effects have been observed for lengths up to five port diameters from the injector. In these regions the boundary layer is disturbed and the ballistic model is not applicable.

There is a real need to study injector effects and to model this process. This will be difficult and perhaps empirical correlations are the most that can be expected. Ideally, it would be desirable to be able to account for oxidizer break-up, oxidizer droplet vaporization and/or combustion rate, droplet trajectories, and interaction of the oxidizer drops with the surface of the grain. Except for the latter, these have been attempted in liquid rocket studies with only limited success. In fact, the inability to accurately predict droplet size distributions produced by injectors remains to be one of the major problems which prevent meaningful quantitative predictions with vaporization-limited combustion instability models. In boost phase operation at high pressures, some liquid oxidizers will be in environments exceeding critical conditions near the head-end of the grain and this can significantly change the required analysis for the consumption rate of the oxidizer drops. The interaction of liquid oxidizers with combusting or decomposing solid surfaces is also not well understood.

The injector effects are directly related to the limiting G_o operations discussed in Section 5.2. Injectors which do not permit adequate distribution and vaporization of the oxidizer may cause local surface flooding with the resulting phenomena discussed above, and in addition, irregular consumption of the solid grain and low performance.

Limited attempts have been made at analytically predicting oxidizer droplet histories in hybrids. LPC has briefly considered the effects of drop size and velocity, and motor geometry on oxidizer vaporization (C13).

Because of the current lack of adequate modeling of injector effects in hybrids, most solutions to injector related problems have been solved experimentally by trial and error. UTC (C4) has used splashblocks of a low regression rate fuel at the head end to prevent severe head-end erosion, to enhance vaporization, and to allow the vaporized oxidizer to enter parallel to the walls of the grain. Uniform regression rates have also been obtained by inserting a diaphragm (orifice, restrictor) near the head end of the grain (17, 22,24).

5.4   Performance

All of the above discussions are of course directly related to obtaining performance from the hybrid rocket. The limits of upper and lower oxidizer mass fluxes and the O/F shift that occurs as a result of throttling were discussed above and must be considered when attempting to obtain maximum performance. In this section several additional considerations are discussed.

Because $\dot{r} \propto G^{0.8} X^{-0.2}$, design of high performing hybrid grain configurations is more complex than for solids. As the grain is consumed G decreases (for a mixed injected mass flow rate) and therefore $\dot{r}$ decreases. If constant thrust and high performance are to be maintained, a progressive burning area will be required that matches the reduction in $\dot{r}$. There is competition between two desirable hybrid features (10). In order to minimize $\dot{r}$ changes with time, changes in G need to be minimized. This necessitates a large initial port size. However, it is also usually desirable to have the maximum volumetric loading possible which in turn means that the port size should be small. A compromise must be reached which depends upon the specific application (10).

To minimize performance losses which result from O/F shifts (due to throttling or grains which are not progressive enough), propellants with flat I_{sp} vs O/F curves should be employed. Reference C6 indicates that in general, low cost propellant systems characteristically have a sharp peak on the I_{sp} vs O/F curve at $(I_{sp})_{max}$, and high density propellants characteristically exhibit the desired flat I_{sp} vs O/F. It has also been reported that the addition of small amounts of AP to the solid grain tends to flatten the I_{sp} vs O/F curve (C6).

Because some of the vaporized fuel is convected downstream below the flame, it has been found necessary to provide mixing of the gaseous products in order to obtain high performance (C4, C5, C13, 24, etc.). Various techniques have been considered and employed. UTC (C4) (and subsequently other investigators) added gas stream turbulators or mixers aft of the grain in an aft plenum region and obtained significant increases in delivered I_{sp}. ONERA (24, etc.) (and subsequently other investigators; (17, etc.)) used a diaphragm (orfice, restrictor) within the grain port and various boundary layer tripers have also been used. The diaphragms and boundary layer trips evidently destroy the turbulent diffusion flame downstream of the device by creating high turbulence which in turn has yielded high regression rates. The heat transfer limited boundary layer model is of little use for internal ballistics calculations in these applications. The well-stirred reactor concept (44, 45, 46, etc.) is in this case probably a better model.

Aft-end oxidizer injection provides mixing and high performance. Multiport grains provide high fuel mass fluxes and provide good mixing at the aft end of the grain. Submerged nozzles have also been used to aid mixing (8).

Reference C13 has shown that although low performance results primarily from poor mixing of the gases, liquid metal on the surface and poor evaporation of the liquid oxidizer also contributed to performance losses. Lieberherr (26) has also discussed the effects of a metallic surface melt on performance.

Aside from the important basic considerations of grain progressivity. mixing of the gases, and maintainence of a constant mixture ratio are several design aspects which may overshadow the utility of the ballistic model(s).

Injector effects, which were discussed above, are one such design aspect and can result in poor fuel utilization, low performance, and unpredictable thrust-time behavior. Throat erosion can also be significant (especially during high pressure, high G_o operation) and can result in significant degradation in performance. The rate of throat erosion is difficult to predict but in many cases it must be establish for incorporation into the ballistics program if meaningful design calculations are to be obtained.

It has been shown (3, 8) that in many cases $\dot{r}$ does not vary appreciably with distance from the head-end. For L/D > 5 the boundary layer edge becomes the port centerline. Oxidizer dilution then occurs but as long as it is not severe (L/D < 25), B has been shown to remain practically constant (5, 8, etc.). The increase in G with X tends to offset the decrease in $\dot{r}$ with X $\left(\dot{r} \propto G^{0.8} B^{.23} X^{-0.2}\right)$. Thus, for the convective heat transfer limited hybrids, $\dot{r}$ does not vary appreciably with X. If the model is to be used for L/D > 25 then B must be considered a variable. In highly radiative systems the shift in O/F due to oxidizer dilution may significantly affect $\dot{r}$.

Because of the numerous variables available to the hybrid designer, past experience will often be required in order to make final decisions on alternate designs. Excellent performance (% theoretical C*, % theoretical I_{sp}, fuel utilization) from hybrid propulsion systems has been reported by numerous investigators (22, C5, C7, C8, C13, etc.).

6.0 COMBUSTION INSTABILITY

Studies of combustion instability in hybrid rockets have been pri-
marily experimental (4, 16, 17, 22, C4, etc.). Occurrences of combustion
instability in hybrids appears to be rather infrequent to date and, when it
has occurred, was normally related to injector failures, inadequate vapor-
ization, etc. Occurrences of acoustic type coupling have been observed only
very rarely. However, with solid oxidizers being added to the grain, opera-
tion in low pressure regimes or with large amounts of oxidizer dilution being
contemplated (ducted rockets, etc.), and with turbulators being used to pro-
mote a well-stirred reactor within the port, it should be expected that the
occurrence of combustion instability will increase in the future. Combustion
instabilities can result in increased regression rates, system overpressuri-
zation, and depending on the type of instability, may either increase or
decrease the combustion efficiency.

Reference 17 has discussed three general types of combustion in-
stability that may occur in hybrid rockets. The first type of instability
is related to the vaporization and combustion of the oxidizer drops and is
normally of low frequency. This type of instability is somewhat related to
that found in vaporization-limited liquid propellant rockets but is probably
more complex. Liquid rocket combustion instability is still not well under-
stood and much debate continues with regard to the mechanism(s) by which the
instability is driven (71, 72). Some models for liquid rocket combustion
instability are concerned with the coupling between the vaporization process,
a pressure disturbance, and the fixed combustor geometry. Instabilities of
this type are normally high frequency acoustic modes. In hybrid rockets,
motor geometry changes with time. The liquid droplets vaporize similar to
that in liquid rockets (although probably much slower) but in most cases the
vapor is not as rapidly consumed. The vapors must be transported by turbu-
lent diffusion to the fuel vapor which is evolving from the grain surface.
Liquid oxidizer may also penetrate the boundary layer and interact with the
grain surface. This may cause periodic localized flooding of the grain with
the resulting pressure spikes discussed above. Elimination of this type of
instability has been accomplished through injector modifications which change
the spray and energy distribution in the port. Reference C4 reports that
elimination of the fuel rich regions in the aft plenum eliminates pressure

fluctuations. Multiport designs were successful in this respect. Moutet and Barrere (22, 23, discussion in 4) have eliminated this type of instability by using a diaphragm in the port as discussed above. This device increases the residence time of the droplets and restricts them to the head-end. Although this device appears to suppress the low frequency instabilities which result from oxidizer droplet vaporization lags, reference 17 reports that it had no suppressing effect upon acoustic modes of instability.

The second type of instability is also of low frequency and many times non-periodic. It is "associated with periodic accumulation and break off of char layers or melted layers (Possibly metallic) at the surface" (17). This phenomenon has been discussed above with reference to low G_o - low $\dot{r}$ operation. This type of instability has been reported by various investigators (C4, C5, C13, etc.). Elimination of this type of instability may require one or more of the following; reduction in metal content, change of binder type, use of aft-mixers, use of nonsymmetic ports, and etc. (4).

The third type of instability is related to solid propellant combustion instability phenomena and could be expected to occur most frequently in hybrids which have kinetically generated pressure sensitive regression rates. Kinetic aspects of hybrid combustion (Section 4.0) are therefore of major importance. There is considerable debate with regards to combustion instability mechanisms in solid propellants (73, etc.) and acoustic mode instability in hybrids will probably be even more difficult to characterize and model. Reference 17 has reported the occurrence of spontaneous instabilities in the longitudinal mode and also possibly in a transverse mode. From the discussion in Section 4.0 it is obvious that many factors may influence acoustic mode instability: oxidizer dilution, solid oxidizer in grain, pressure level, port configuration, mass flux through the port, and etc.

The model developed by Marksman, et al (17) for the steady-state regression rate pressure sensitivity was discussed above (Section 4.0). That investigation was primarily concerned with modeling acoustic mode combustion instability in hybrid rockets. Their experiments indicated among other things that (a) the metal content of the solid grain did not affect the pressure sensitivity of the regression rate or the frequency of

the instability, (b) binder pyrolysis characteristics did not affect the regression rate pressure sensitivity, and (c) use of more reactive oxidizer resulted in less pressure sentitivity.

Only limited experimental and analytical (7, 17, 18, 37) investigations have been conducted in this area and much additional study is required.

The use of hybrid rockets for tactical weapon systems may necessitate studying acceleration effects for both the liquid oxidizer droplet behavior and the regression rate of the solid grain. Both of these areas have received only brief consideration (C11, C13).

7.0   SUMMARY OF AREAS OF APPLICATION OF
HEAT TRANSFER LIMITED INTERNAL BALLISTICS MODEL

The internal ballistic model developed by United Technology Center is found to be applicable--

- With very good accuracy to propellant systems which are convective heat transfer limited and which have relatively simple grain geometries. Minimum subscale testing is required.

- With good accuracy to propellant systems which are heat transfer limited but with both convective and radiative contributions to the total heat flux. Subscale testing is required to evaluate empirical constants.

In both cases the models are limited to hybrid operation between the upper and lower critical mass fluxes. Scaling of hybrid combustors has been successfully demonstrated (12, C5, C6, C10). Throttling applications are readily handled unless transient phenomena are of long duration. The major items which overshadow the calculations made with the ballistics model are injector effects and throat erosion.

In addition, the model can be used--

- With lesser accuracy and scaling reliability for systems which have slightly pressure sensitive regression rates as a result of kinetic effects. Additional empirical parameter required in model. Additional subscale testing is required.

- Probably with decreased accuracy for some air-augmented hybrids and solid fuel ramjets if care is used in defining the mass-flux employed in the model and if regression rates are not kinetically controlled.

8.0 SUMMARY OF REQUIRED AREAS OF INVESTIGATION

1. Basic investigations are needed in the area of turbulent transport mechanisms in boundary layers with combustion and high rates of wall mass addition.

2. Experimental and analytical investigations of combustion instability in hybrids are required. These are required in order to determine whether or not combustion instability will actually be a future problem in hybrid combustion. Studies of the regression rate and oxidizer vaporization rate responses to pressure and velocity disturbances are required.

3. Directly related to combustion instability studies are required studies of the kinetic aspects of hybrid combustion. This remains a controversial subject and will have increased importance as more advanced hybrid systems (AP added to grain, etc.) are considered.

4. Hybrid ignition is a current problem that requires immediate attention.

5. Related to ignition phenomena are studies required on injector ballistics effects. This is also a current problem. These studies will probably have to be empirical in nature.

6. Radiation heat transfer in hybrids (and in air-augmented systems) requires further study. The current simplified approach used in the ballistics model may not suffice for higher energy propellant systems which may have significantly increased radiation transport.

7. Investigations are required to more adequately determine the applicability of the current model to air-augmented hybrids and solid fuel ramjets.

8. Improvements can be made in the current ballistics model (variable mixture ratio and radiation temperature, unvaporized oxidizer effects, etc.).

1. Caras, G. J., "Hybrid Propulsion: A Bibliography," NASA-TM-X-60024, 1967.

2. Marxman, G. A., And Gilbert, M., "Turbulent Boundary Layer Combustion in the Hybrid Rocket," Ninth Symposium (International) on Combustion, Academic Press, New York, 1963, p. 371.

3. Marxman, G. A., Wooldridge, C. E., and Muzzy, R. J., "Fundamentals of Hybrid Boundary Layer Combustion," Heterogeneous Combustion, Progress in Astronautics and Aeronautics, Vol 15, Academic Press, New York, 1964, p. 485.

4. Marxman, G. A., and Wooldridge, C. E., "Research on the Combustion Mechanism of Hybrid Rockets," Advances in Tactical Rocket Propulsion, AGARD Cp No. 1, Aug 1968, p. 421.

5. Marxman, G. A., "Combustion in the Turbulent Boundary Layer on a Vaporizing Surface," Tenth Symposium (International) on Combustion, The Combustion Institute, Pittsburgh, 1965, p. 1337.

6. Marxman, G. A., and Wooldridge, C. E., "The Incompressible Turbulent Boundary Layer on a Flat Plate with Homogeneous Injection," Technical Memorandum TM-33-64-U1, United Technology Center, Sunnyvale, 1964.

7. Marxman, G. A., "Boundary Layer Combustion in Propulsion," Eleventh Symposium (International) on Combustion, The Combustion Institute, Pittsburgh, 1967, p. 269.

8. "Investigation of Fundamental Phenomena in Hybrid Propulsion (U)," United Technology Center, Sunnyvale, California, Vol I (U), Final Technical Report, No. UTC 2097-FR, Contract Number w 64-0659-c, November 1965.

9. Wooldridge, C. E., and Muzzy, R. J., "Boundary Layer Turbulence Measurements with Mass Addition and Combustion," AIAA J. 4, 2009-2016, (1966).

10. Wooldridge, C. E., and Muzzy, R. J., "Internal Ballistic Considerations in Hybrid Rocket Design," J. Spacecraft and Rockets 4(2), 255-262, 1967.

11. Wooldridge, C. E. and Muzzy, R. J., "Measurements in a Combusting Turbulent Boundary Layer with Porous Wall Injection," Tenth Symposium (International) on Combustion, Combustion Institute, Pittsburgh, Pa., 1964, pp. 1351-1362.

12. Muzzy, R. J., "Demonstration of a High Thrust Hybrid Thrust Chamber Assembly," Final Technical Report (Part II) of Contract AF 04(611)-11618, United Technology Center, April 1968, AFRPL-TR-68-56, Part II.

13. Muzzy, R. J., "Schlieren and Shadowgraph Studies of Hybrid Boundary-Layer Combustion," AIAA Journal, 1:2159, 1963.

14. "Injector Effects on the Ballistics of Hybrid Propulsion Systems," UTC Planning Document 70-13, March 1970.

15. "Low-Signature Hybrid Demonstration Program," UTC Technical Proposal, 8 May 1967.

16. Wooldridge, C. E., and Marxman, G. A., "Combustion Instability and the Role of Chemical Kinetics in Hybrid Combustion," AIAA Paper 68-498.

17. Wooldridge, C. E., Marxman, G. A., and Kier, R. J., "Investigation of Combustion Instability in Hybrid Rockets," Stanford Research Institute, NASA CR-66812.

18. Wooldridge, C. E., Marxman, G. A., "A Simple Regression Rate Model for the Pressure Sensitive Domain of Hybrid Combustion," WSS/CI Paper No. 68-22.

19. "Advanced Development Program For a Tactical Air-Launched Hybrid," Planning Document 69-13, UTC, 12 June 1969.

20. Barrere, M. and Moutet, A., "La Propulsion par Fusees Hybrides," XIV International Astronautical Congress, Oct 1963, pp. 213-249.

21. Moutet, A. and Barrere, M., "Contribution a L'etude de la Combustion dans les Fusees a Lithergol ou Hybrides," Advances in Aeronautical Sciences, Pergamon Press, New York, 1961, pp. 465-495.

22. Moutet, A., "Hybrid Rocket Propulsion," Advances In the Astronautical Sciences, Vol. 19, pp. 911-931.

23. Moutet, A., "Research on Hybrid Propellants," [Researches sur les Propulseurs Hybrides] presented to the Journees d"Etude Sur la Propulsion Chimique, Paris, May 1964. (A65-18870).

24. Barrere, M. and Moutet, A., "Liquid-Solid Rockets," International Science and Technology, Aug 1967, p. 64-74.

25. Barrere, M., "L'Allumage Des Fusees Hybrids (Hybrid Rocket Ignition)," La Recherche Aerospatiale, 104, Jan-Feb, 1965.

26. Lieberherr, J. F., "Melting Phenomena and Surface Reactions in Hybrid Propellant Combustion," AGARD CP No. 53, Reactions Between Gases and Solids, Oct. 1969.

27. Smoot, L. D., and Price, C. F., "Regression Rate Mechanism of Nonmetalized Hybrid Fuel Systems," AIAA J. 3, 1408-1413, 1965.

28. Smoot, L. D., and Price, C. F., "Regression Rates of Metalized Hybrid Fuel Systems," AIAA J. 4, 810-915, (1966).

29. Smoot, L. D., and Price, C. F., "Pressure Dependence of Hybrid Fuel Regression Rates," AIAA J. 5, 102-106, (1967).

30. Houser, T., "Final Report, Research in Hybrid Combustion" Rocketdyne Report R-5179, May 1963.

31. Houser, T. J., and Peck, M. V., "Research in Hybrid Combustion," Progress in Astronautics and Aeronautics: Heterogeneous Combustion, edited by H. G. Wolfhard, I. Glassman, and L. Green Jr., Academic Press, Inc., New York, 1964, Vol. 15, p. 559.

32. Peck, M. V. et al., "Research in Hybrid Combustion," Rocketdyne Summary Report R-3446, January 1962.

33. Kosdon, F. J., and Williams, F. A., "Pressure Dependence of Nonmetalized Hybrid Fuel Regression Rates," AIAA J. 5, 774-778 (1967).

34. Williams, F. A., "Theory of Propellant Ignition by Heterogeneous Reaction," AIAA Journal, Vol 4, Aug 1966, p. 1354-1357.

35. Williams, F. A., "Response of a Burning Fuel Plate to Sound Vibrations," AIAA Journal, Vol. 3, Nov 1965, p. 2112-2123.

36. Williams, F. A., "Grain Design and Throttling of Hybrid Rocket Motors," Chemical Engineering Progress Symposium Series, No. 61, Vol. 62.

37. Williams, F. A., "Application of Unsteady Boundary Layer Flow Theory to the Prediction of Oscillatory Stability Limits for Combustion in Liquid and Hybrid Rocket Motors," AIAA paper No. 65-41, AIAA 2nd Aerospace Sciences Meeting, New York, Jan 1965.

38. Stickler, D. B., and Kumar, R. N., "Polymer Degradation Rate Control of Hybrid Rocket Combustion," MIT Space Propulsion Laboratory, SPL Report No. 70-2, Sep 1970.

39. Stickler, D. B. and Kumar, R. N., "Polymer Degradation Theory of Pressure Sensitive Hybrid Combustion," To be published in 13th Symposium (International) on Combustion.

40. Searle, H. D., "Measurement of Radiant Heat Transfer to the Fuel Grain Surface in a Hybrid Rocket using Thin Film Resistance Thermometers," Aeronautical Engineer Thesis, Naval Postgraduate School, 1966.

41. Bruntlett, C. E., "A Parametric Study of the Radiant Energy Flux to the Fuel Surface During Hybrid Combustion," M.S. Thesis in Aeronautical Engineering, Naval Postgraduate School, Dec 1967.

42. Miller, E., "Hybrid Rocket Combustion Regression Rate Model AIAA J. 4, 752-753 (1966).

43. Fineman, S., "Some Analytical Considerations of the Hybrid Rocket Combustion Problem," Master's Thesis, Princeton University (1962).

44. Courtney, W. G., Kineyko, W. R., and Dawson, B. E., "Combustion During Prependicular Flow" Progress in Astronautics and Aeronautics: Heterogeneous Combustion, Academic Press Inc., 1964, pp. 523-558.

45. Avery, W. H. and Hart, R. W., "Combustor Performance with Instantaneous Mixing," Ind. & Eng. Chem. 45, p. 1634, 1953.

46. Longwell, J. P., Frost, E. E., and Weiss, M. A., "Flame Stability in Bluff Body Recirculation Zones, Ind. & Eng. Chem. 45, p. 1629, 1953.

47. Penner, S. S., "Chemical Rocket Propulsion and Combustion Research," New York, Gordon & Breach, 1962. pp. 145-153.

48. Rabinovitch, B., "Regression Rates and the Kinetics of Polymer Degradation," Tenth Symposium (International) on Combustion. The Combustion Institute, 1965, pp. 1395-1404.

49. Lengelle, G., "Thermal Degradation Kinetics and Surface Pyrolysis of Vinyl Polymers," AIAA Journal, Vol. 8, No. 11, Nov 1970.

50. Saraniero, M. A., Caveny, L. H., Summerfield, M., "Combustion Anomalies in Stop Restart Firing of Hybrid Rocket Engines," Aerospace and Mechanical Sciences Report No. 945, NASA Grant NGR 31-001-109, Princeton University, Sep 1970.

51. Hurt, L. J. and Anderson, S. E., "Hydrodynamic Factors Influencing the Combustion of Hybrid Propulsion Systems," Chemical Engineering Progress Symposium Series, No. 61, Vol. 62.

52. Drew, W. B., "An Analytical Method for Determining the Melt-Layer Thickness and Regression Rates for a Hybrid or Tribrid Rocket Motor," NASA, TND-5699, May 1970.

53. Lees, L., "Convective Heat Transfer with Mass Addition and Chemical Reactions," Combustion and Propulsion, Third AGARD Colloquium, Pergamon Press, New York, 1958.

54. Green, L., Jr., "Introductory Considerations on Hybrid Rocket Combustion," Progress in Astronautics and Aeronautics: Heterogeneous Combustion, edited by H. G. Wolfhard, I. Glassman, and L. Green, Jr., Academic Press Inc., New York, 1964, Vol. 15, pp. 451-484.

55. Spangler, William E., "A Study of the Transport Processes in a Hybrid Rocket Engine," Ph.D. Thesis, Tulane University, New Orleans, Louisiana, August 1965.

56. Chen, T. N. and Toong, T. Y., "Laminar Boundary Layer Wedge Flows with Evaporation and Combustion," Progress in Astronautics and Aeronautics: Heterogeneous Combustion, Academic Press Inc., 1964, p. 643.

57. Friedlander, S. K., and Keller, K. H., "The Structure of the Zone of Diffusion Controlled Reaction," Chem Eng Sci 18, 365-375 (1963).

58. Vinckier, C. and Van Tiggelen, A., "Burning Rates of Polymethylmetacrylate in Mixtures with Ammonium Perchlorate or in Hybrid Systems when Mixed with Ammonium Chloride," Astronautica Acta, Vol. 14, March 1969, p. 261-265.

59. Shun-i-chi Tsuge' and Yoshito Fujiwara, "Fine Structure of Hybrid Combustion", Astronautica Acta, Vol 14, April 1969, p 327-335.

60. Cohen, N. S., "Combustion Considerations in Fuel-Rich Propellant Systems," AIAA Journal, 7 July 1969, p 1345-1352.

61. Mickley, H. S. and Davis, R. S., "Momentum Transfer for Flow over a Flat Plate with Blowing," NACA TN-4017, 1957.

62. Zabelka, R. J. and Brink, D. F., "Studies in Hybrid Rocket Combustion," Final Report, Jet Propulsion Center, Purdue University, 1964.

63. Denison, M. R. and Baum, E., "A Simplified Model of Unstable Burning in Solid Propellants" ARS J., 31, 1112-1122, 1961.

64. Karlovitz, "Combustion Waves in Turbulent Gases," Vol II, High Speed Aerodynamics & Jet Propulsion, Series, Princeton U. Press, Princeton, 1956.

65. Osman, R. V., "An Experimental Investigation of A Lithium Aluminum Hydride-Hydrogen Peroxide Hybrid Rocket," Chemical Engineering Progress Symposium Series, No. 61, Vol 62.

66. Schlichting, H., Boundary Layer Theory, 4th Edition, McGraw-Hill.

67. Henderson, C. B., Graham, P. H., and Robinson, C. N., "A Comparison of Reaction Rate Models for the Fracture of Solids," International Journal of Fracture Mechanics, Vol. 6, No. 1, March 1970.

68. Zhurkov, S. N., "Kinetic Concept of the Strength of Solids," International Journal of Fracture Mechanics, Vol 1, No. 4, Dec 1965.

69. Tobolsky, A. and Eyring H., "Mechanical Properties of Polymeric Materials," Journal of Chemical Physics, Vol II, March 1943.

70. Leeming, H., "Failure Behavior of Highly Filled Rubber Based Propellants," ICRPG Mechanical Behavior Working Group, 6th Meeting, CPIA Publication No. 158, Oct 1967.

71. Strahle, W. C., "New Considerations on Causes for Combustion Instability in Liquid Propellant Rockets," Comb. Sci. & Tech. 2, 1970, pp 29-40.

72. Netzer, D. W. and Osborn, J. R., "Investigation of Combustion Characteristics of a Bi-Phase Rocket," Purdue University, Jet Propulsion Center, Report No. TM 68-2, April 1968.

73. Price, E. W., "Comments on Role of Aluminum in Suppressing Instability in Solid Propellant Rocket Motors," AIAA J., May 1971.

74. Satoaki, Omori, "Hybrid Combustion with Surface Pyrolysis," Ph.D. Thesis, Tokyo University, June 1970.

75. Price, E. W., Bradley, Jr., H. H., Dehority, G. L., and Ibiricu, M. M., "Theory of Ignition of Solid Propellants," AIAA J. 4, No. 7, 1153-1181.

CONFIDENTIAL REFERENCES

(unclassified titles)

C1. Boyland, R. E., "Hybrid Rocket Performance," Report No. ADR-01-08-62.2, Gruman Aircraft Engineering Corporation, 28 Nov 1962.

C2. "Investigation of Fundamental Phenomena in Hybrid Propulsion," Final Technical Report No. UTC 2007-FR, Contract No. NOW 61-1000-C, UTC, May 1963.

C3. "Investigation of Fundamental Phenomena in Hybrid Propulsion," Final Technical Report No. UTC 2051-FR, Contract No. 63-0737-C, UTC, July 1964.

C4. "Investigation of Fundamental Phenomena in Hybrid Propulsion," Vol II, Final Technical Report Contract NOW 64-0659-C, UTC 2097-FR, Nov 1965.

C5. "Demonstration of a High Thrust Hybrid Thrust Chamber Assembly," Final Technical Report AFRPL-TR-68-56, Part I, UTC 2219-FR, April 1968.

C6. Wooldridge, C. E. and Bae, W. E., "The Comprehensive Design of Practical Hybrid Propulsion Systems," paper presented at the ICRPG/AIAA Solid Propulsion Conference, Washington, D. C., July 1966.

C7. "Demonstration of Hybrid Engine Operation Characteristics for Tactical Weapons," Final Report No. UTC 2292-FR, UTC, 26 June 1968.

C8. "Hybrid Tactical Operations Development," Final Report No. UTC 2319, UTC, Feb 1969.

C9. Feemster, J., "Low-Cost Tactical Hybrid Propulsion System," Final Report UTC 2351-FR, UTC, Feb 1970.

C10. Muzzy, R. J., Feemster, J. R., and Spangler, W. E., "The Design of a Hybrid Rocket Propulsion System," Proceedings of the Ninth Liquid Propulsion Symposium, Vol. I, CPIA, St. Louis, Oct 1967.

C11. Matthews, D. R., "Effect of Spin on Hybrid Rocket Internal Ballistics," UTC Final Report, NASA-CR-66298, 1967.

C12. "Fundamental Investigation of Hypergolic Ignition of Solid Propellants," UTC Report No. 2074-FR, Two Volumes, 1966.

C13. Smoot, L. D., and Price, C. F., "Hybrid Propulsion Research Program," Final Report of Contracts DA04495, ORD-3577, and DA04495 AMC218(z), Lockheed Propulsion Company Report No. 640-F, Two Volumes, Aug 1965.

C14. Smoot, L. D., Price, C. F., and Taylor, D. E., "Regression Rate Mechanisms of Metalized Hybrid Fuel Systems," AIAA 6th Solid Propellant Combustion Conference, Feb 1-3, 1965.

C15. "Feasibility Demonstration of Hybrid Propulsion Systems," Lockheed Propulsion Company, Final Technical Report, Contract No. DA-04-495-AMC-1632(z), Aug 1966.

C16. Mead, F. B., Jr. and Bornhorst, B. R., "Certification Tests of Hybrid Propulsion System for the Sandpiper Target Missile," Technical Report No. AFRPL-TR-69-73, Air Force Rocket Propulsion Laboratory, June 1969.

DOCUMENT CONTROL DATA - R & D		
(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)		

1. ORIGINATING ACTIVITY (Corporate author)	2a. REPORT SECURITY CLASSIFICATION
Chemical Propulsion Information Agency JHU/Applied Physics Laboratory 8621 Georgia Avenue Silver Spring, MD 20910	UNCLASSIFIED
	2b. GROUP

3. REPORT TITLE

Hybrid Rocket Internal Ballistics

4. DESCRIPTIVE NOTES (Type of report and, inclusive dates)

5. AUTHOR(S) (First name, middle initial, last name)

David W. Netzer, Naval Postgraduate School, Monterey, CA
Warren E. Bae (Editor),

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
January 1972	45 plus i-ix	91

8a. CONTRACT OR GRANT NO. N00017-62-C-4401	9a. ORIGINATOR'S REPORT NUMBER(S)
b. PROJECT NO. NAVAIR-330, Job Order No. 4505-1519,	
c. 045011-01	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report) none
d.	

10. DISTRIBUTION STATEMENT

Approved for Public Release;
distribution unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Naval Air Systems Command (AIR-330) Department of the Navy Washington, DC 20360

13. ABSTRACT

This report presents a summary of hybrid rocket internal ballistics. Discussions
are primarily concerned with the current theory that is based on a heat transfer
limited model. Areas of application, major limitations, kinetic effects, and
controversial aspects are briefly discussed. Areas are identified that require
additional analytical and/or experimental investigation.

The review was completed in June 1971, and does not include bibliographic material
after that date.

DD FORM 1473 (PAGE 1)
1 NOV 65

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Combustion Instability						
Combustion Modeling						
Hybrid Rockets						
Ignition						
Internal Ballistics						
Regression rate						

U14536